skin effect. Its purpose is essentially to develop an analytical method for solving such problems, because the analytical solution is useful in the treatment of non-linear problems as we shall see later.

Dynamical variations of the temperature can also be examined within the framework of this theory. These generalizations are currently under investigation and the results will be reported elsewhere.

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Forced convection heat transfer in smooth tubes roughened by helically coiled ribbons

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INTRODUCTION

WITH INCREASING emphasis on economic energy saving considerations, efforts are being made to develop better heat transfer surfaces to produce more efficient heat exchange equipment. Internal roughness such as sand-grain textures [1, 2], internal ribbing [3, 4] and spirally corrugated tube surfaces [5] have been studied or applied with varying degrees of success.

Helically coiled wires [6] or ribbons, fitted tightly inside smooth tubes, give a considerable increase in heat transfer rate without a significant increase in friction power, as these tubes produce some helical flow at the periphery of flow, superimposed upon the main axial flow, and thus influence the velocity distribution, the turbulence level and the turbulent wall shear. As no previous study has been made on smooth tubes, roughened with coiled ribbons, the present investigation was carried out to study their frictional and heat transfer performance and develop suitable correlations for momentum and heat transfer roughness functions, based on friction and heat transfer similarity laws.

THEORETICAL BACKGROUND

Nikuradse [1] used the law of the wall concept and obtained the turbulent flow velocity distributions for the smooth and rough tubes

$$u^+ = 2.5 \ln y^+ + 5.5 \quad \text{(smooth tube)} \tag{1}$$

$$u^+ = 2.5 \ln (y/h) + R(h^+) \quad \text{(rough tube).} \tag{2}$$

On integration over the tube cross-section, equation (2) gives

$$R(h^+) = \sqrt{(2/f)} + 2.5 \ln (2h/D) + 3.75. \tag{3}$$

For the fully rough region ($h^+ > 70$), $R(h^+)$ attained a constant value of 8.48 for the sand-grain rough tubes of Nikuradse [1] and Dipprey and Sabersky [2]. Results of turbulent friction factors expressed as $R(h^+)$ were successfully correlated by Webb *et al.* [3] for tubes with transverse ribs, and by Ganeshan and Raja Rao [5] for spirally corrugated tubes.

Dipprey and Sabersky [2] first developed a heat transfer similarity law, analogous to the friction similarity law and correlated their heat transfer results in terms of Prandtl number and roughness Reynolds number

$$G(h^+, Pr) = \left[\left(\frac{f}{2St} - 1 \right) \Big/ \sqrt{(f/2)} + R(h^+) \right] = 5.19 (Pr)^{0.44} (h^+)^{0.20}. \tag{4}$$

The recent work of Ganeshan and Raja Rao [5] and Gee and Webb [4] indicates that the heat transfer similarity law can be applied to other rough surfaces, having discrete two-dimensional roughness elements.

EXPERIMENTAL WORK

The three helically coiled ribbons were fabricated by winding a long strip of copper sheet (0.72 mm thick and 4.5 mm wide) on a cylindrical rod of 23.5 mm diameter, using a precision lathe. Thin line impressions of the desired helical paths were engraved on the rods, before the winding operations. The pitch of the coiled ribbons used was 41, 21 and 11 mm, corresponding to helix angles of 51°, 66° and 79°, respectively. A cut section of the smooth tube, roughened by a typical coiled ribbon is shown in Fig. 1, and geometrical properties of the tubes are listed in Table 1. The ribbon coils, when introduced into the smooth tube, fitted tightly, ensuring close contact between the ribbon surface and tube wall.

The apparatus for this work is the same as used in our earlier study [6], and consisted of a 2050 mm long double-pipe heat exchanger, along with auxiliary equipment for circulation of hot (test) liquid on the tube side, and cold water on the annulus side, in closed loops. Water and 40% aqueous glycerol were used as the test liquids.

NOMENCLATURE

A area [m^2]
C_p specific heat [kJ kg^{-1} K^{-1}]
D tube diameter [m]
h ribbon thickness [m]
h_i, h_o heat transfer coefficients [W m^{-2} K^{-1}]
k thermal conductivity [W m^{-1} K^{-1}]
L length [m]
p pitch [m]
ΔP pressure drop [N m^{-2}]
Q heat duty [W]
R tube radius [m]
T temperature [°C]
u point velocity [m s^{-1}]
u^* friction velocity [m s^{-1}]
U overall coefficient [W m^{-2} K^{-1}]
V mean velocity [m s^{-1}]
w coiled ribbon width [m]
x tube wall thickness [m]
y distance from wall [m].

Dimensionless groups
f friction factor, $D\Delta Pg_c/2\rho V^2L$
$G(h^+, Pr)$ heat transfer roughness function, $(f/2St)-1/\sqrt{f/2}+R(h^+)$
h^+ roughness Reynolds number, hu^*/ν
Nu Nusselt number, (hD/K)
Pr Prandtl number, $(C_p\mu/K)$
Re Reynolds number $(DV\rho/\mu)$
$R(h^+)$ momentum transfer roughness function, $\sqrt{(2/f)}+2.5\ln(2h/D)+3.75$
St Stanton number $(h_i/C_pV\rho)$
u^+ dimensionless velocity (u/u^*)
y^+ dimensionless distance (yu^*/ν).

Greek symbols
α helix angle [deg]
μ dynamic viscosity [Pa s]
ν kinematic viscosity [m^2 s^{-1}]
ρ density [kg m^{-3}].

Subscripts
a augmented tube
b bulk condition
i inside
lm logarithmic mean
m metal wall
o equivalent smooth tube value
w wall condition.

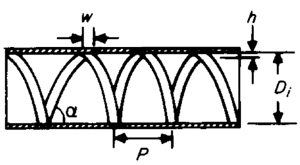

FIG. 1. Ribbon coil roughened tube: sectional view.

An isothermal pressure drop at 27°C was first measured over a test section length of 2470 mm for turbulent flow of water and 40% glycerol inside the smooth tube and three ribbon-roughened tubes, using U-tube manometers, with mercury and carbon tetrachloride as manometric liquids. Heat transfer runs were conducted for turbulent flow cooling of hot water and 40% glycerol, flowing on the tube side, using cooling water flowing at a constant flow rate, in turbulent flow, in the annulus of the exchanger. Steady-state flow rates of the hot and cold fluids were obtained from the respective calibrated rotameters, and inlet and outlet temperatures of each stream were measured by mercury thermometers accurate to ± 0.1°C. The desired hot test liquid inlet temperature (55 ± 1°C) was secured by controlling the variacs, connected to the three tubular immersion heaters (total capacity 13 kW), equipped in the test liquid storage tank.

The maximum measurement errors were found to be ± 2.0% in flow rate, ± 2.5% in pressure drop and ± 3.0% in temperatures. The uncertainties in the calculated friction factor and heat transfer coefficient are ± 4.6 and 6.0%, respectively.

Table 1. Geometrical properties of ribbon-roughened tubes: $D_i = 25.0$ mm, $h = 0.72$ mm, $w = 4.5$ mm.

Tube	D_e (mm)	h/D_e	p (mm)	p/D_e	α (deg)
Smooth (s)	25.0	0	—	—	—
1	23.6	0.0305	41	1.74	51
2	22.7	0.0317	21	0.93	66
3	21.5	0.0335	11	0.51	79

RESULTS AND DISCUSSION

Friction factor

Fanning's friction factor was calculated from

$$f = D\Delta pg_c/2\rho V^2L. \quad (5)$$

The smooth tube turbulent flow friction factors measured for water were in good agreement with the Blasius equation, as seen in Fig. 2

$$f = 0.079Re^{-0.25} \quad (6)$$

in the range $5000 < Re < 25{,}000$, and this served the purpose of calibration of the experimental setup. In view of the changing cross-section of the ribbon-roughened tube, the volumetric diameter, as defined below, was used as the equivalent diameter, D_e

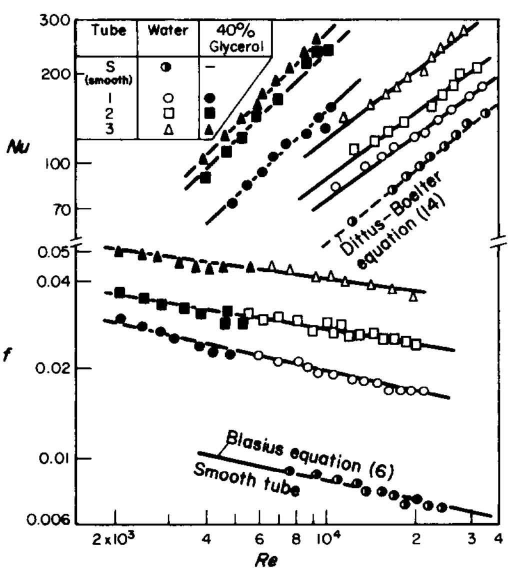

FIG. 2. Variation of f and Nu with Re.

$$D_e = \frac{(4 \times \text{free volume for flow per unit length})}{(\text{area of wetted surface per unit length})} \quad (7)$$

in the calculation of Re and f. The f–Re plot, shown in Fig. 2, reveals that friction factors in the ribbon-roughened tubes were 2.2–5 times higher than for the smooth tube, and the data for each tube were fitted well by straight lines, over the range of Reynolds numbers, 2000–20,000, thus indicating early onset of turbulent flow in these tubes. Further, increased coiled ribbon helix angle (or decreased pitch) resulted in reduced dependence of f on Re—owing to markedly increasing surface roughness. The friction factors were correlated as a function of Re as follows:

$$f = 0.115Re^{-0.19} \quad (\text{tube 1}; \alpha = 51^\circ) \quad (8)$$

$$f = 0.118Re^{-0.16} \quad (\text{tube 2}; \alpha = 66^\circ) \quad (9)$$

$$f = 0.132Re^{-0.13} \quad (\text{tube 3}; \alpha = 79^\circ). \quad (10)$$

An analysis of friction results expressed as a roughness momentum transfer function $R(h^+)$ reveals that $R(h^+)$ values as shown in Fig. 3 are vertically displaced, due to the geometrically dissimilar nature of surface roughness of the tubes ($15.3 < p/h < 56.9$), and moreover $R(h^+)$ shows a rising trend with (h^+), as observed for spirally corrugated tubes. Values of $R(h^+)$, when corrected for the influence of $[h/(p-w)]$ as recommended by Ganeshan and Raja Rao [5] were correlated satisfactorily with the following equation:

$$R(h^+)\left(\frac{h}{p-w}\right)^{0.52} = 0.273 \ln(h^+) + 0.127. \quad (11)$$

The results for water in tube 1 were, however, consistently lower by 10–15%, perhaps owing to the difficulty of obtaining perfect contact of this coiled ribbon of higher pitch ($p \neq 41$ mm) with the tube wall, especially at higher Reynolds numbers.

Heat transfer

The overall heat transfer coefficient U_i was calculated from

$$U_i = Q/(A_i \Delta T_{lm}). \quad (12)$$

Since the ribbon is thin ($h = 0.72$ mm), and it makes contact with the tube wall, the additional surface offered by the lateral sides of the ribbon is only 3.5–13.0% of the tube wall area. However, the inside surface of tube wall A_i is used for all the tubes. From preliminary heat transfer runs with a smooth tube heat exchanger, the outside cooling water heat transfer coefficient h_o was obtained from the Wilson plot ($1/U_i$ vs $1/V_i^{0.8}$). The value of h_o was constant (at 3950 $\text{W m}^{-2}\text{K}^{-1}$) in view of the essentially constant flow rate and mean temperature of the cold water used in the annulus of the heat exchanger.

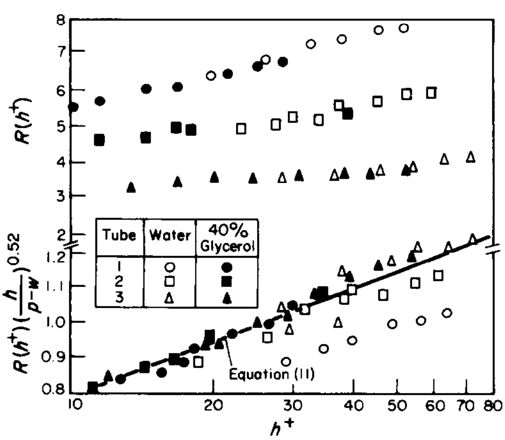

FIG. 3. Variation of $R(h^+)$ with (h^+).

As all heat transfer runs with ribbon-roughened tubes were conducted using constant flow rate and mean temperature of the cooling water in the annulus, corresponding to those used in the smooth tube heat exchanger, a value of h_i for the test liquid was obtained from the measured U_i and known h_o ($= 3950\ \text{W m}^{-1}\text{K}^{-1}$) using the equation

$$\frac{1}{h_i} = \left(\frac{1}{U_i} - \frac{D_i}{h_o D_o} - \frac{x D_i}{K D_m}\right). \quad (13)$$

As the inner and outer surfaces of the tube wall were clean, dirt film resistances were assumed negligible. The h_i values computed from equation (13) ranged from 2000 to 8000 $\text{W m}^{-2}\text{K}^{-1}$ for water ($Pr = 3.45$) and from 1600 to 5200 $\text{W m}^{-2}\text{K}^{-1}$ for 40% glycerol ($Pr = 13.8$).

Nusselt numbers ($h_i D_i/K$) obtained from h_i are shown as a function of Reynolds number in Fig. 2. The smooth tube values agreed closely with those calculated from the Dittus–Boelter equation for cooling runs of the present work

$$Nu = 0.023Re^{0.8}\,Pr^{0.3} \quad (\text{smooth tube: cooling runs}). \quad (14)$$

Values of Nu for the three ribbon-roughened tubes (Fig. 2) were 1.3, 1.6 and 2.0 times the smooth tube values for water ($10{,}000 < Re < 25{,}000$), while those for 40% glycerol ($3000 < Re < 12{,}000$) were 1.4, 2.0 and 2.2 times greater than for the smooth tube. It is clear that smooth tubes, roughened with coiled ribbons of higher helix angles of 66° and 79°, gave better thermal performance for the more viscous 40% glycerol. The following correlations were proposed for the Nusselt numbers in the ribbon-roughened tubes.

Tube	Water	40% glycerol
1	$Nu = 0.031Re^{0.8}\,Pr^{0.3}$	$Nu = 0.0096Re^{0.97}\,Pr^{0.03}$ (15)
2	$Nu = 0.037Re^{0.8}\,Pr^{0.3}$	$Nu = 0.0125Re^{0.99}\,Pr^{0.3}$ (16)
3	$Nu = 0.050Re^{0.8}\,Pr^{0.3}$	$Nu = 0.0132Re^{0.99}\,Pr^{0.3}$ (17)

Metal wall temperatures (T_w) estimated for typical runs varied from 36 to 41°C depending upon Reynolds number and Prandtl number. Thus (T_b/T_w) ranged from 1.30 to 1.15, and $(\mu_b/\mu_w)^{0.14}$ was found to vary from 0.96 to 0.99, thereby establishing the negligible effect of the viscosity correction factor for the two test liquids of the present work ($Pr = 3.45$; 13.8).

The heat transfer roughness function $G(h^+)$, was calculated from values of f, St and $R(h^+)$ using equation (4) and the $G(h^+)$ vs (h^+) plot is shown in Fig. 4. The data points for 40% glycerol cover essentially the transition region ($5 < h^+ < 30$), whereas those for water cover the fully rough region ($h^+ > 30$). When $G(h^+)$ was corrected for the influence of Prandtl number and $(h/(p-w))$ through the use of $Pr^{-0.55}$ and $(h/(p-w))^{0.18}$ as suggested [5] the values of the G-function in the fully rough region ($h^+ > 30$), ranged from 10 to 15 only in agreement with the expected trends for other roughened surfaces like spirally corrugated tubes and sand-grain roughened tubes. It is clear from Fig. 4 that coiled ribbon-roughened tubes, offering microroughness at the wall, may give intermediate performance between sand-grain roughened tubes [2] and wire-coil inserted tubes [6].

Performance evaluation

Tube performance was evaluated, using criterion 3 of Bergles *et al.* [7], which aims to increase the heat duty for the constraints of constant pumping power and constant basic geometry. The performance ratio, R_3 was evaluated as (U_a/U_0) and an examination of Fig. 5 showing R_3 vs Re_0 (equivalent smooth tube Reynolds number) reveals that

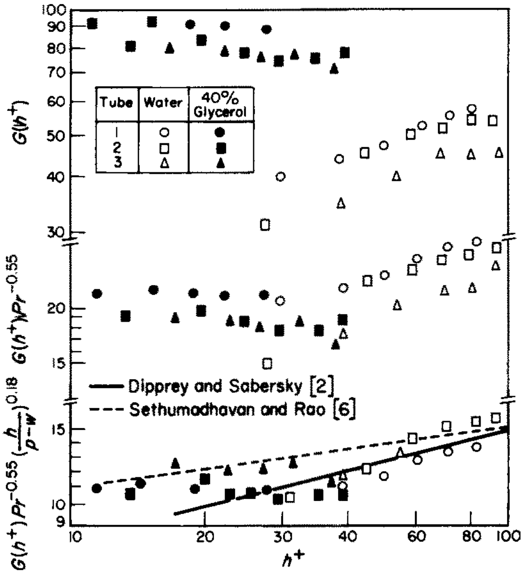

FIG. 4. Variation of $G(h^+)$ with (h^+).

enhancement in heat duty for water ranged from 15 to 30% only for tubes 1 and 2, but it varies from 35 to 45% for tube 3. However, for 40% glycerol, tubes 2 and 3 give 35–50% enhanced heat duty and tube 1 gives only 10–20%. Obviously tube 3 (with a coiled ribbon pitch of 11 mm) gives the best performance compared to other tubes, and it gives an increased heat duty of 45 and 50% for water and 40% glycerol, respectively, around a Reynolds number of 15,000–20,000.

CONCLUSIONS

Turbulent flow friction factors in smooth tubes, roughened by helically coiled ribbons are 2.2–5.0 times greater, and Nusselt numbers are 1.3–2.2 times higher than those for the smooth tube over the range, $3000 < Re < 25{,}000$, and correlations are proposed for f and Nu.

Friction and heat transfer results were also analysed in terms of roughness momentum and heat transfer functions, and suitable correlations are suggested for $R(h^+)$ and $G(h^+)$.

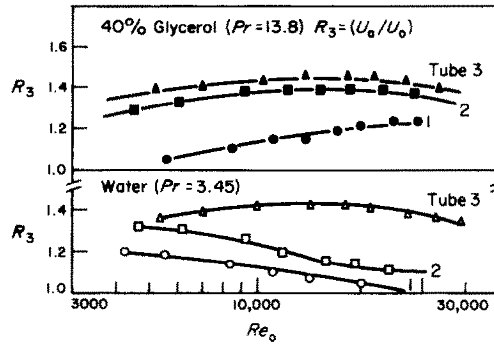

FIG. 5. Variation of R_3 with Re_o.

Tube performance was evaluated on the basis of heat duty per unit pumping power, and tube 3 (p = 11 mm, α = 79°) giving 45–50% enhanced heat duty was identified as the most efficient tube for water and 40% glycerol used as test liquids.

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A combined convection correlation for vertical downward cooling flow in a natural circulation loop

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INTRODUCTION

SEVERAL empirical and analytical correlations for combined free- and forced-convection flow through vertical tubes without connection to a natural circulation loop have been derived [1, 2]. Holman and Boggs [3] derived an empirical correlation for the combined convection of upward turbulent flow in a vertical heating tube of a natural circulation loop. It can be seen from their results that the correlation is quite different from that of a single heater and a pure forced convection. A similar phenomenon was also observed by Creveling *et al.* [4] for the cooling jacket in a toroidal natural circulation loop and by Bau and Torrance [5] for the heater